The impact of underwater acoustic channel structure and dynamics on the performance of adaptive coherent equalizers

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Abstract. Channel estimate based equalizers are those for which observations of the received signal are used to estimate the channel impulse response and possibly the statistics of the interfering noise field, and these estimates are used to calculate the equalizer filter coefficients. Channel estimate based decision feedback equalizers (CE-DFE), linear MMSE equalizers (L-MMSE), and Passive Time-Reversal equalizers (P-TR) are all examples of coherent channel estimate based equalizers. Equations are derived for analyzing the performance of these channel estimated based equalizers. The performance is characterized in terms of the mean squared soft decision error (σ_s^2) of each equalizer. This error is decomposed into two components. These are the minimum achievable error (σ_o^2) and the excess error (σ_ε^2). The former is the soft decision error that would be realized by the equalizer if the filter coefficient calculation were based upon perfect knowledge of the channel impulse response and statistics of the interfering noise field. The latter is the additional soft decision error that is realized due to errors in the estimates of these channel parameters. Here, the impact of errors in the channel impulse estimation errors on σ_ε^2 is considered (i.e., the equalizer is assumed to have accurate knowledge of the statistics of the interfering noise field.). The error equations allow for a direct comparison of the performance characteristics of these equalizers as well as an evaluation of the impact of the characteristics of the acoustic channel on equalizer performance.

INTRODUCTION

Adaptive coherent equalizers can be divided into two classes, the first being direct adaptation equalizers and the second being channel estimate based equalizers. Direct adaptation equalizers are those for which the filter coefficients of the equalizer are directly adjusted based upon observations of the received signal. Channel estimate based equalizers are those for which observations of the received signal are used to estimate the channel impulse response and possibly the statistics of the interfering noise field and these estimates are used to calculate the equalizer filter coefficients. Channel estimate based decision feedback equalizers (CE-DFE), linear MMSE equalizers (L-MMSE), and Passive Time-Reversal equalizers (P-TR) [3] are all examples of coherent channel estimate based equalizers. The use of the expressions derived here allows these three equalizers to be evaluated in the following context: that is, given estimates of the channel impulse response and the statistics of the interfering noise fields, how do the three different methods of computing equalizer filter coefficients impact the equalizer performance?

The format of this paper is as follows. The next section outlines notation as well as the expressions for the modeled channel impulse response and the equalizer filter coefficients. This section develops insights that allows the equalizer filtering problem to be thought of in terms of "replica vectors" that are derived from the channel impulse response. The section following that presents expressions for the soft decision error achieved by each of the three types of equalizers. The expressions are used to develop insight into the characteristics of channels that most significantly impact equalizer performance, and the relative performance characteristics of the three types of equalizers are evaluated. Finally, the performance equations for a particular type of channel estimator (the exponentially weighted least squares estimator) are presented and that equation are used to evaluate a required error correlation matrix.

CHANNEL AND EQUALIZER MODEL

The acoustic channel is modeled as a time-varying, discrete time system described by the complex baseband input delay-spread function[1] (IDSF) [1]. The received signal at time n is given by

$$u[n] \;=\; \tilde{\mathbf{g}}^h[n]\tilde{\mathbf{d}}[n]+v[n]. \tag{1}$$

where

$$\tilde{\mathbf{g}}[n]\stackrel{\triangle}{=}[g[n,N_c-1],\cdots,g[n,0],\cdots,g[n,-N_a]]^t \text{ , and}$$

$$\tilde{\mathbf{d}}[n]\stackrel{\triangle}{=}[d[n-N_c+1],\cdots,d[n],\cdots,d[n+N_a]]^t$$

are samples of the IDSF and transmitted data symbols, respectively. $d[n]$ is the transmitted data symbol at time n, $v[n]$ is the interfering noise at time n, $g[n,m]$ is the IDSF for delay m at time n, the superscript t denotes transpose, and the superscript h denotes Hermitian. The quantities N_a and N_c denote, respectively, the number of acausal and causal taps in the IDSF[2].

Throughout this paper, lower case letters denote scalar quantities, lower case bold face letters denote vectors (all vectors are assumed to be column vectors), and upper case bold face letters denote matrices. Herein the received signal is assumed to be sampled at the transmit symbol rate. The extension of the analysis to fractionally spaced systems is conceptually straight forward, but the notation is cumbersome. The final results of the analysis are equally applicable to symbol rate and fractionally spaced systems.

The equalizers considered here each consist of a finite impulse response (FIR) *feed-forward* filter that filters the received signals and, in the case of the CE-DFE, an FIR feedback filter that filters and feeds back estimates of the transmitted data symbol.[3] The

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[1] The input delay-spread function is one form of what is more commonly referred to as the time-varying impulse response

[2] Physical underwater acoustic channels are all causal. However, it is sometimes conceptually convenient to think of some point in delay (the variable m) within the IDSF to be the zero delay tap and those points that preceed this point in the delay variable to be acausal taps.

[3] The development herein assumes a single channel equalizer. The extension to a multichannel equalizer utilizing a receive array is straightforward and does not alter the results of the analysis.

output of the filter is the soft decision estimate, $\hat{d}_s[n]$, of the transmitted data symbol, $d[n]$. The estimate $\hat{d}_s[n]$ is the input to a decision device that generates the final estimate, $\hat{d}[n]$, of the transmitted data symbol.

For a linear equalizer (e.g., the L-MMSE and P-TR equalizers) the soft decision estimate of the transmitted data symbol, $\hat{d}_s$, is given by

$$\hat{d}_s[n] \;=\; \mathbf{h}^h[n]\,\mathbf{u}[n], \tag{2}$$

where $\mathbf{h}[n]$ is a vector of the feedforward filter coefficients at time n and

$$\mathbf{u}[n] \stackrel{\triangle}{=} [u[n-L_c-1],\cdots,u[n],\cdots,u[n+L_a]]^t\,. \tag{3}$$

Here, L_c and L_a denote the number of causal and acausal taps, respectively, of the feedforward filter. The notation $\mathbf{h}_{\mathrm{lin}}$ and $\mathbf{h}_{\mathrm{tr}}$ will be used to denote the filter coefficient vectors for the L-MMSE and P-TR equalizers, respectively. For the CE-DFE, $\hat{d}_s$ is given by

$$\hat{d}_s[n] \;=\; \mathbf{h}_{\mathrm{ff}}^h[n]\,\mathbf{u}[n] \;+\; \mathbf{h}_{\mathrm{fb}}^h[n]\hat{\mathbf{d}}_{fb}[n]. \tag{4}$$

Here, $\mathbf{h}_{\mathrm{ff}}$ and $\mathbf{h}_{\mathrm{fb}}$ are vectors of the coefficients of the CE-DFE feedforward and feedback filters, respectively. For a feedback filter of length L_{fb} symbols, $\hat{\mathbf{d}}[n]$ is a vector of estimates of past transmitted data symbols given by $\hat{\mathbf{d}}_{fb}[n] \stackrel{\triangle}{=} \left[\hat{d}[n-L_{fb}],\cdots,\hat{d}[n-1]\right]^t$.

For both the linear and decision feedback equalizers, $\mathbf{u}[n]$ is the received signal vector that is used by the feedforward filter to generate the soft decision estimate. Combining (1) and (3) yields

$$\mathbf{u}[n] \;=\; \mathbf{G}[n]\,\mathbf{d}[n] \;+\; \mathbf{v}[n], \tag{5}$$

where

$$\mathbf{d}[n] \stackrel{\triangle}{=} [d[n-L_c-N_c+2],\cdots,d[n],\cdots,d[n+L_a+Na]]^t$$

and

$$\mathbf{v}[n] \stackrel{\triangle}{=} [v[n-L_c],\cdots,v[n],\cdots,v[n+L_a]]^t\,.$$

$\mathbf{G}[n]$ is the sampled IDSF matrix with the i^{th} row composed of $\tilde{\mathbf{g}}^h[n-L_c+i]$ packed with leading and trailing zeros to position it the appropriate columns of the matrix with respect to the elements of the vector $\mathbf{d}[n]$. It is instructive to represent $\mathbf{G}$ using its column vectors indexed in the following manner:

$$\mathbf{G}[n] \;=\; \left[\mathbf{g}_{(N_c+L_c-2)},\cdots,\mathbf{g}_1,\mathbf{g}_0,\mathbf{g}_{-1},\cdots,\mathbf{g}_{-(N_a+L_a)}\right], \tag{6}$$

The dependence of the columns of $\mathbf{G}[n]$ on the time index n is suppressed here for notational convenience. The vector $\mathbf{g}_i$ is a replica vector for the data symbol $d[n-i]$ in the received signal vector $\mathbf{u}[n]$. Partition the transmit data symbols in $\mathbf{d}[n]$ into three groups: $\mathbf{d}_{fb}[n] \stackrel{\triangle}{=} \left[d[n-L_{fb}],\cdots,d[n-1]\right]^t$, $d[n]$, and $\mathbf{d}_o[n]$ which is composed of the remaining elements of $\mathbf{d}[n]$. Partition the columns of $\mathbf{G}[n]$ into three similarly defined sets: $\mathbf{G}_{fb}$, $\mathbf{g}_0$, and $\mathbf{G}_o$. Then (5) can be rewritten as

$$\mathbf{u}[n] \;=\; \mathbf{g}_0 d[n] \;+\mathbf{G}_{fb}\mathbf{d}_{fb}[n] \;+\; (\mathbf{v}[n]+\mathbf{G}_o\mathbf{d}_o[n]). \tag{7}$$

The first term is the portion of the received signal vector, $\mathbf{u}[n]$, that corresponds to the transmitted data symbol to be estimated, $d[n]$. The second term is the portion of $\mathbf{u}[n]$ that can be cancelled by the output of the feedback filter in a CE-DFE, and the terms in the parenthesis represent an effective observation noise that the feedforward filter must try to eliminate. Assuming that the data sequence is a zero mean, white sequence with a variance of one[4], the data sequence is independent of the channel IDSF and $v[n]$, and that $\mathbf{v}[n]$ is a zero mean sequence with covariance $\mathbf{R}_v$ that is independent of the channel IDSF, the effective noise correlation matrix, $\mathbf{Q}$, can be written as

$$\mathbf{Q} \;=\; \mathbf{R}_v \;+\; \mathbf{G}_o\mathbf{G}_o^h. \tag{8}$$

With the model and quantities so defined, a number of approaches can be used to calculate the optimal filter coefficients. One such approach is given in [2]. In that paper, the effective noise correlation matrix, denoted with the symbol $\mathbf{R}$, includes the impact of channel estimation errors. Therefore, the calculated filter coefficients and subsequent error analysis are valid for the case where the DFE has accurate knowledge of both the noise statistics and the second order statistics of the channel estimation errors. For the filter calculation and performance analysis presented here, there is no assumption that the DFE knows the statistics of the channel estimation errors.

The filter coefficients for the three equalizers are calculated using estimated quantities for $\mathbf{R}_v$ and $\mathbf{G}$. In the following expressions, these estimated quantities are denoted by the hat (e.g., $\hat{\mathbf{R}}_v$). The filter coefficient vectors for the L-MMSE and CE-DFE equalizers are selected to minimize the mean squared soft decision error (E$[\mid \hat{d_s}[n]-d[n] \mid^2]$ assuming that the estimates of $\mathbf{R}_v$ and $\mathbf{G}$ are accurate and that the statistical assumptions stated in the paragraph before (8) hold. The expressions for these filter coefficient vectors are

$$\mathbf{h}_{\text{ff}} \;=\; \frac{\hat{\mathbf{Q}}^{-1}\hat{\mathbf{g}}_0}{1+\hat{\mathbf{g}}_0^h\hat{\mathbf{Q}}^{-1}\hat{\mathbf{g}}_0}, \quad \mathbf{h}_{\text{fb}} \;=\; -\hat{\mathbf{G}}_{fb}^h\mathbf{h}_{\text{ff}}, \quad \text{and} \tag{9}$$

$$\mathbf{h}_{\text{lin}} \;=\; \frac{(\hat{\mathbf{Q}}+\hat{\mathbf{G}}_{fb}\hat{\mathbf{G}}_{fb}^h)^{-1}\hat{\mathbf{g}}_0}{1+\hat{\mathbf{g}}_0^h(\hat{\mathbf{Q}}+\hat{\mathbf{G}}_{fb}\hat{\mathbf{G}}_{fb}^h)^{-1}\hat{\mathbf{g}}_0}. \tag{10}$$

The P-TR equalizer is a normalized matched filter so its coefficients are given by

$$\mathbf{h}_{\text{tr}} \;=\; \frac{\hat{\mathbf{g}}_\mathbf{0}}{\hat{\mathbf{g}}_0^h\hat{\mathbf{g}}_0}. \tag{11}$$

EQUALIZER PERFORMANCE

The performance is characterized in terms of the mean squared soft decision error $\left(\sigma_s^2 \triangleq \text{E}[\mid \hat{d_s}[n]-d[n] \mid^2]\right)$ of each equalizer. This expectation is conditioned upon the estimate of the channel IDSF. This error is decomposed into two components. These are

[4] The assumption of unit variance can be made without any loss of generality of the results.

the minimum achievable error (σ_o^2) and the excess error (σ_ε^2) such that $\sigma_s^2 = \sigma_o^2 + \sigma_\varepsilon^2$. σ_o^2 is the soft decision error that would be realized by the equalizer if the filter coefficient calculation were based upon perfect knowledge of the channel impulse response and statistics of the interfering noise field. σ_ε^2 is the additional soft decision error that is realized due to errors in the estimates of these channel parameters.

The Minimum Achievable Error

For the three different equalizers, the minimum achievable error is given by

$$\sigma_{o_{dfe}}^2 = \frac{1}{1 + \hat{\mathbf{g}}_0^h \hat{\mathbf{Q}}^{-1} \hat{\mathbf{g}}_0}, \tag{12}$$

$$\sigma_{o_{lin}}^2 = \frac{1}{1 + \hat{\mathbf{g}}_0^h (\hat{\mathbf{Q}} + \hat{\mathbf{G}}_{fb} \hat{\mathbf{G}}_{fb}^h)^{-1} \hat{\mathbf{g}}_0}, \text{ and} \tag{13}$$

$$\sigma_{o_{tr}}^2 = \frac{\hat{\mathbf{g}}_{\mathbf{0}}^h (\hat{\mathbf{Q}} + \hat{\mathbf{G}}_{fb} \hat{\mathbf{G}}_{fb}^h) \hat{\mathbf{g}}_{\mathbf{0}}}{\hat{\mathbf{g}}_0^h \hat{\mathbf{g}}_0}. \tag{14}$$

Comparing equations (12), (13), and (14), it can be shown that

$$\sigma_{o_{dfe}}^2 \leq \sigma_{o_{lin}}^2 \leq \sigma_{o_{tr}}^2.$$

Furthermore, it can be shown that $\sigma_{o_{dfe}}^2$ and $\sigma_{o_{line}}^2$ will always decrease when the number or received signal channels or the length of the feedforward or feedback filters is increased.

Note that the minimal achievable error for the DFE is completely characterized by the quadratic product of the replica vector associated with the data symbol being estimated, $\mathbf{g}_{(0)}$, and the inverse of the effective noise correlation matrix, $\mathbf{Q}$, which represents the contribution of the observation noise and the acausal data symbols ($d[m]$ for $m > n$) and causal symbols $d[m]$ for which $m < n - L_{fb}$ to the input of the feedforward filter and the soft data estimate. Thus, the minimal achievable error of the DFE is determined by the projection of the replica vector $\mathbf{g}_{(0)}$ on both the observation noise correlation matrix $\mathbf{R}_\nu$ and the replica vectors corresponding to the acausal data symbols and the causal symbols not canceled by the output of the feedback filter. The structure of the channel IDSF impacts the minimal achievable error through these replica vectors. Note that in non-minimum phase channels where there will be a large number of replica vectors corresponding to acausal channel taps, the minimal achievable error will tend to larger than in minimum phase channel with comparable delay spreads. An example of one such class of non-minimum phase channels is the long range, deep water channel.

The Excess Error

The impact of error in the estimation of channel parameters on the performance of each coherent equalizer is quantified by the excess error, σ_ε^2. Here, the impact of errors in the estimation of the IDSF is considered. Let the true channel IDSF matrix, $\mathbf{G}[n]$ be given by

$$\mathbf{G}[n] = \hat{\mathbf{G}}[n] + \mathbf{E}_G \tag{15}$$

where $\mathbf{E}_G$ is the error in the estimate of the IDSF matrix. For analytic simplicity, assume further that $\mathbf{E}_G$ is statistically independent of $\hat{\mathbf{G}}[n]$ and has zero mean. Then,

$$\sigma^2_{\varepsilon_{dfe}} \;=\; \mathbf{h}^h_{\mathrm{ff}} \mathbf{R}_{E_G} \mathbf{h}_{\mathrm{ff}}, \tag{16}$$

$$\sigma^2_{\varepsilon_{lin}} \;=\; \mathbf{h}^h_{\mathrm{lin}} \mathbf{R}_{E_G} \mathbf{h}_{\mathrm{lin}}, \;\text{ and} \tag{17}$$

$$\sigma^2_{\varepsilon_{tr}} \;=\; \mathbf{h}^h_{\mathrm{tr}} \mathbf{R}_{E_G} \mathbf{h}_{\mathrm{tr}}, \tag{18}$$

where $\mathbf{R}_{E_G} \stackrel{\triangle}{=} \mathrm{E}[\mathbf{E}_G \mathbf{E}^h_G | \hat{\mathbf{G}}]$. Thus the sensitivity of each equalizer to channel IDSF estimation errors is determined by the magnitude squared of the vector of the equalizer's feedforward filter coefficients and the projection of these coefficient vectors on the eigenstructure of $\mathbf{R}_{E_G}$. For the region of high adaptive processing gain, that is $\hat{\mathbf{g}}^h_0 \hat{\mathbf{Q}}^{-1} \hat{\mathbf{g}}_0 >> 1$, and for equalizers with the same feedforward filter length, it can be shown that

$$|\, \mathbf{h}_{\mathrm{ff}} \,|^2 > |\, \mathbf{h}_{\mathrm{lin}} \,|^2 > |\, \mathbf{h}_{\mathrm{tr}} \,|^2 \,.$$

Thus, if $\mathbf{R}_{E_G}$ is a scalar times the identity matrix and the equalizers have the same number of taps in their feedforward filters, the P-TR equalizer will be less sensitive to channel estimation errors than either the L-MMSE or CE-DFE equalizers. The following section describes conditions under which $\mathbf{R}_{E_G}$ will meet this requirement.

Equation (16) is counterintuitive in that the excess error does not appear to depend upon either the feedback filter coefficients or the errors in these coefficients. This result can be explained as follows. Combining equations (4) and (7) results in

$$\hat{d}_s[n] \;=\; \mathbf{h}^h_{\mathrm{ff}}[n]\; (\mathbf{g}_0 d[n] \;+ \mathbf{G}_{fb} \mathbf{d}_{fb}[n] \;+\; \mathbf{v}[n] + \mathbf{G}_o \mathbf{d}_o[n]) \;+\; \mathbf{h}^h_{\mathrm{fb}}[n] \hat{\mathbf{d}}_{fb}[n].$$

Substituting in (9) for $\mathbf{h}_{\mathrm{fb}}[n]$, (15), and rearranging terms yields

$$\hat{d}_s[n] \;=\mathbf{h}^h_{\mathrm{ff}}[n]\; (\hat{\mathbf{g}}_0 d[n] + \mathbf{v}[n] + \hat{\mathbf{G}}_o \mathbf{d}_o[n]) + \mathbf{h}^h_{\mathrm{ff}}[n]\; (\hat{\mathbf{G}}_{fb} \mathbf{d}_{fb}[n] - \hat{\mathbf{G}}_{fb} \hat{\mathbf{d}}_{fb}[n]) + \mathbf{h}^h_{\mathrm{ff}}[n] \mathbf{E}_G \mathbf{d}[n].$$

The expected value magnitude squared of the difference between the first term and the actual data symbol, $d[n]$, is the minimum achievable error. The second term is the only term that depends on the feedback filter coefficients. Assuming that the past symbol decisions are accurate (i.e., $\hat{\mathbf{d}}_{fb}[n] = \mathbf{d}_{fb}[n]$, this term equals zero. The third term represents the excess error. The expected value of the magnitude squared of this term equals (16). Assuming that the channel estimation error is independent of the channel estimate, the transmitted data, and the observation noise, this equals the excess error, $\sigma^2_{\varepsilon_{dfe}}$.

CHANNEL ESTIMATION ERROR

To gain insight into the structure of $\mathbf{R}_{E_G}$ and the impact of channel structure and dynamics on the error in estimating the channel IDSF, consider the performance characteristics of the commonly used Exponentially Weighted Least Squares algorithm. With this algorithm, the estimate of the channel IDSF is given by

$$\hat{\tilde{\mathbf{g}}}[n] \;=\; \arg\min_{\mathbf{g}} \sum_{m=-\infty}^{n} \lambda^{(n-m)} \mid u[m] - \mathbf{g}^h \tilde{\mathbf{d}} \mid^2,$$

where λ is a constant "forgetting factor" between zero and one. Assume that the channel IDSF, $\tilde{\mathbf{g}}[n]$, is a zero mean, wide sense stationary random process with correlation matrix $\mathbf{R}_{\tilde{\mathbf{g}}}[m] \stackrel{\triangle}{=} \mathrm{E}\left[\tilde{\mathbf{g}}[n]\,\tilde{\mathbf{g}}^h[n+m]\right]$. Then, the error correlation matrix $\mathbf{R}_{\varepsilon} \stackrel{\triangle}{=} \mathrm{E}\left[\left(\hat{\tilde{\mathbf{g}}}[n] - \tilde{\mathbf{g}}[n+1]\right)\left(\hat{\tilde{\mathbf{g}}}[n] - \tilde{\mathbf{g}}[n+1]\right)^h\right]$ is given by

$$\mathbf{R}_{\varepsilon} \;=\; \frac{2}{(1+\lambda)}\mathbf{R}_{\tilde{\mathbf{g}}}[0] - \frac{1}{(1+\lambda)}\sum_{m=0}^{\infty}\left(\frac{\lambda^m}{W}\right)\left(\mathbf{R}_{\tilde{\mathbf{g}}}[m+1] \;+\; \mathbf{R}_{\tilde{\mathbf{g}}}^h[m+1]\right) + \frac{1}{(1+\lambda)}\left(\frac{\sigma_v^2}{W}\right)\mathbf{I},$$

where $W \stackrel{\triangle}{=} \sum_{m=0}^{\infty} \lambda^m = (1-\lambda)^{-1}$. To relate this result to the matrix $\mathbf{R}_{E_G}$ used in (16), (17), and (18), note that $\mathbf{R}_{E_G}$ is well approximated by a Toeplitz matrix. Furthermore, the elements along the i^{th} diagonal of $\mathbf{R}_{E_G}$ are equal to the sum of the elements along the i^{th} diagonal of $\mathbf{R}_{\varepsilon}$. This will be used in the following section to develop intuition regarding the impact of channel dynamics on equalizer robustness.

For wide sense stationary, uncorrelated scattering (WSSUS) channels, $\mathbf{R}_{\varepsilon}$ is a diagonal matrix thus resulting in $\mathbf{R}_{E_G}$ equaling the trace of $\mathbf{R}_{\varepsilon}$ times the identity matrix. Therefore, evaluation of (16), (17), and (18) for this case shows that the excess error for each equalizer equals the trace of $\mathbf{R}_{\varepsilon}$ times the magnitude squared of the feedforward filter coefficient vector for each equalizer. This result is independent of the distribution of the IDSF estimation error among the taps of the IDSF vector. While these correlation matrices are not conditioned upon the channel estimate (or equivalently, the calculated feedforward filter weights) as required to properly evaluate (16) through (18), they do lend insights into the channel and equalizer characteristics that impact robustness with respect to channel estimation errors.

CONCLUSION

Expressions for the components of the soft decision error for linear MMSE, decision feedback, and passive time-reversal equalizers have been presented. The error is decomposed into a minimum achievable error and the excess error. Evaluation of the expressions shows that the ranking of the three equalizers from best to worst in terms of minimum achievable error is the CE-DFE, the L-MMSE equalizer, and the P-TR equalizer. However, with simplifying assumptions it is shown that the P-TR equalizer is the least sensitive of the three to channel estimation errors. Insights are presented into the channel characteristics that most significantly impact equalizer performance.

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