Modeling Shallow Water Propagation With Scattering From Rough Boundaries

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Abstract. Results of PE simulations at 3 kHz will be described for propagation in a shallow water waveguide with a rough sea surface and a flat water-sediment interface overlying absorbing sediments. Pressure fields are simulated in two space dimensions and have been obtained using a wide-angle PE code that accounts for scattering from a rough sea surface. The PE simulation approach is a Monte Carlo method, requiring solutions for many surface realizations in order to obtain results for field moments. To obtain a fast, yet accurate model for average field quantities, a transport theory method based on coupled modes has been developed for both the first and second moments of the field. Estimates for the fourth moment of the field have also been obtained based on the transport theory results for the first and second moments. Transport theory results for the coherent and total intensity, and for the scintillation index, will be presented and compared with Monte Carlo simulations.

INTRODUCTION

Modeling of shallow water propagation can be challenging because of the need to treat 3-D spatial and temporal variations of the sound speed field, spatial bathymetric variations, and multiple boundary interactions for the propagating field. In practice, the problem is typically made even more difficult by an incomplete knowledge of the environmental conditions. Here we focus on the effects of rough surface scattering at sea surface boundary interactions, and for simplicity, take the sound speed to be constant within the water (1,500 m/s) and within the seafloor sediment (1,600 m/s), and assume the water column has a constant average depth (50 m) to a flat water-sediment interface. In addition, the field in the homogeneous sediment is attenuated at 0.5 dB/λ, and the sediment-to-water density ratio is 2.0.

For the problem described, and for typical surface roughness conditions, the modeling task is simpler at low frequencies (< 1 kHz), because the propagating field is dominated by the coherent field, and the surface scattered incoherent component is relatively small by comparison. (The coherent field, or the first moment of the field, is given by the average of the complex field over an ensemble of rough surface realizations.) In this case, one could proceed iteratively by first obtaining in a self-consistent manner the coherent field as a function of range and depth, and then using the coherent field to obtain the scattered field [1-3]. As the acoustic frequency increases, however, the scattered incoherent field becomes larger relative to the coherent field, and it becomes more important to treat the effects of multiple scattering in forward propagation. We consider a frequency of 3 kHz where simulations to be described shortly show that the incoherent and coherent fields are of comparable magnitude for ranges up to about 5 km, while at longer ranges the coherent field dominates. To obtain "ground truth" for the effects of surface scattering on shallow water propagation at 3 kHz, we have used numerical simulations based on rough surface PE. In addition, we have developed a fast, yet accurate method based on transport theory for modeling both the coherent field and the average total intensity of the field that applies for all relative levels of the incoherent field. Results for the scintillation index have also been obtained.

RESULTS

The propagation simulations were done using a wide-angle PE method developed by Rosenberg [4] that we believe accurately accounts for forward scattering from a rough sea surface (in two space dimensions). The Rosenberg propagation model is an extension to a wide-angle PE propagation model developed by Collins [5]. Rough sea surface realizations generated for use in the simulations are consistent with a one-dimensional cut through a two-dimensional isotropic spectrum of a Pierson-Moskowitz form [6]. For examples shown, the surface waves are produced by a wind speed of 7.7 m/s (15 knots). A point source is located at the mid depth of 25 m, and a vertical beam pattern has been applied with a full width of 20° and with the beam center aimed up at a 10° grazing angle. The simulations have been done for a CW source.

PE simulation results for the coherent intensity (first moment of the field) and the average total intensity (second moment of the field) based on 50 surface realizations are shown in Fig. 1. The simulation is done in two space dimensions, and cylindrical divergence is not included. The color bar denotes the field intensity in dB. At short range the total intensity is noticeably greater than the coherent intensity due to the presence of the incoherent field scattered from the rough sea surface. At longer range the total intensity approaches the coherent intensity, except in regions where the coherent intensity has nulls. These trends can be understood qualitatively from both ray and normal mode perspectives [7]. Monte Carlo simulations of this type give reliable results but are computer intensive and time consuming. It would, therefore, be advantageous to have a more practical method that yields these average results without depending on a Monte Carlo approach.

We have developed a transport theory method for computing directly the first, second, and fourth moments of the propagating field. Space does not permit a detailed exposition of the method here, and these details will be presented elsewhere. Instead, we briefly indicate the basic ideas, and then show how well the transport theory results agree with those obtained with rough surface PE. The starting point is to expand the field in unperturbed normal modes and then obtain the evolution equations for the mode amplitudes, accounting for mode coupling due to scattering from a particular realization of the rough surface. Small surface height perturbation theory is used to evaluate the mode coupling terms. Stopping at this stage would yield an alternative Monte Carlo simulation method. Transport theory gives evolution equations for moments of the mode amplitudes at the cost of some approximations. The evolution equations for the first moment of the mode amplitudes are obtained by formally averaging the set of mode amplitude equations and using transport theory approximations as given, for example, by Van Kampen [8]. The final result is a first-order evolution equation of form

$$\frac{dA}{dx} = RA \tag{1}$$

for the N-component vector A of average mode amplitudes with R an $N \times N$ matrix. The solution to (1) is given by

$$A(x) = \exp(Rx)A(0). \tag{2}$$

For the second moment, one first obtains the evolution equation for all products of two mode amplitudes with one amplitude complex conjugated. Formally averaging and making transport theory approximations again leads to equations of the form of (1) and (2), but now A is an N^2-component vector and R is an $N^2 \times N^2$ matrix.

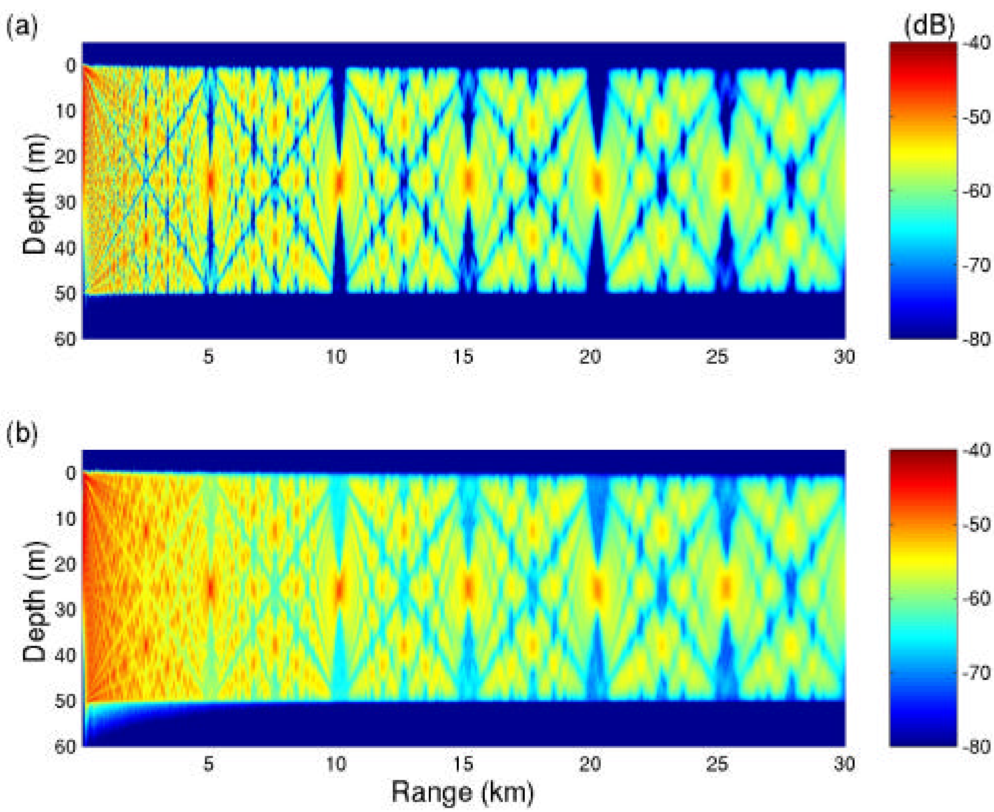

FIGURE 1. PE simu lation for coherent intensity (a) and total intensity (b) obtained by averaging over the results for 50 rough surface realizations.

For our problem there would be 74 discrete modes (or "propagating modes") plus continuum modes if the sediment were modeled as a lossless, infinite half-space. In practice, we have taken the computational region to consist of a 50 m water layer and a 150 m sediment layer; this converts the continuum modes into a set of closely spaced discrete modes. (Because sediment attenuation is also included, some of the continuum modes are promoted to discrete modes; this will be described elsewhere.) We have investigated in detail the number of modes that must be retained in transport theory to obtain convergence with PE results. This was done by comparing transport theory and PE results for mode amplitude decay with range, using both the average mode amplitudes (the first moments) and the average of the absolute squares of the mode amplitudes (a subset of the second moments). The PE mode amplitudes were obtained by projecting the PE fields onto the mode functions as a function of range for each rough surface realization and then performing averages over realizations of the complex mode amplitudes and their absolute squares. We found that true convergence between transport theory and PE results could be obtained by taking $N = 74$, the total number of propagating modes, each of which corresponds to rays that reflect from the bottom at grazing angles below the critical angle (about 20°). However, to obtain this convergence, we also found it necessary to extend sums over modes that occur in the matrix elements of R up to 200. This means that coupling terms are included that couple energy from below to above the critical angle, where it would be rapidly lost into the bottom; this energy loss needs to be included. Since we do not transport modes that correspond to rays that are above the critical angle, this energy is effectively removed from the problem at the point the coupling occurs. However, this has almost the same effect as transporting these higher modes, since they would be rapidly attenuated by absorption in the bottom. Thus, restricting the transported modes to the propagating modes turns out to be a good approximation when internal matrix element sums are extended to sufficiently high values (200 in our case).

Obtaining the solution for the second moment with Eq. (2) would be entirely impractical for an N as large as 74. Fortunately, a major simplification can be made with essentially no loss in accuracy: we assume the effect of cross-mode coherences on the mode intensities can be neglected in Eq. (1). Our rationale is that these effects should largely average out over relatively short ranges due to range dependence of the phases on the right-hand side. We refer to this as the Dozier-Tappert approximation, since the same approximation was made in their transport theory treatment of acoustic propagation through internal waves [9, 10]. However, we do not neglect cross-mode coherence in constructing the intensity field; rather, we assume only that the incoherent contributions to the cross-mode coherence vanish. In the Dozier-Tappert approximation for the second moment, A becomes an N-component vector and R becomes an $N \times N$ matrix. We have implemented transport theory with and without the Dozier-Tappert approximation, and found that for our problem the Dozier-Tappert approximation is perfectly adequate and yields excellent agreement with rough surface PE results. We make a related approximation for the first moment, which reduces R to diagonal form, and which we have also found to be highly accurate by comparing results with and without the approximation. Though R becomes diagonal, coherent mode amplitude decay due to rough surface scattering is still accurately taken into account through internal sums over modes in the diagonal elements that we extend up to 200. In what follows, the Dozier-Tappert approximation will be taken to imply the approximations described for both the first and second moments of the field; this approximation has been used for all transport results shown.

Transport theory results for the coherent intensity and the total intensity are shown in Fig. 2. A comparison of the intensities from PE (Fig. 1) with the corresponding transport theory predictions shows remarkable agreement. The coherent intensities in part (a) of these two figures are difficult to distinguish. (The difference above the mean surface occurs simply because the domain for the PE simulations extends well above the mean surface though the coherent field vanishes there, while the domain for the transport method terminates at the mean surface and the plot has not been modified to show a vanishing field above the mean surface.) The average total intensities (coherent plus incoherent) in part (b) of the figures are also in very good agreement in the water column.

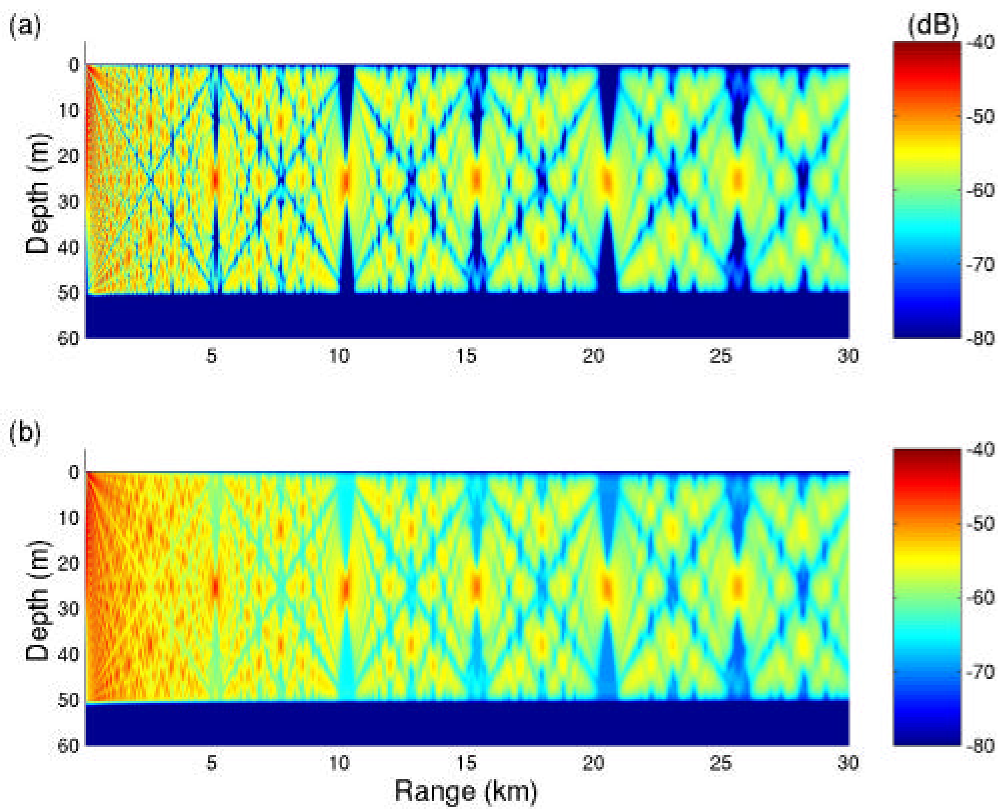

FIGURE 2. Transport theory results for coherent intensity (a) and total intensity (b) obtained in the Dozier-Tappert approximation.

Note that the PE simulation shows scattered intensity in the sediment at short range that is not present in the transport theory plot. This difference follows from our restriction to propagating modes with transport theory and has two elements. First, we begin by projecting the starting field onto the set of 74 propagating modes. However, the starting field employed has some initial energy propagating at angles relative to the horizontal that are greater than the critical angle. This energy is rapidly lost into the bottom in the PE simulation, but is not included in the transport model; differences downrange due to this should be negligible. Second, rough surface scattering continually promotes energy from discrete propagating modes to modes that are rapidly attenuated in the bottom (the set of closely spaced discrete modes that represent the continuum modes plus the "promoted" discrete modes). Or, equivalently, scattering continually transfers energy from below to above the critical angle, and this energy shows up in the PE simulation as it is being lost into the bottom. As discussed previously, in our transport method this energy effectively vanishes in the water column at the point it scatters to angles above the critical angle, and therefore does not appear as intensity in the sediment. Since this energy loss is being properly taken into account, differences downrange should again be negligible. If the field in the sediment were of primary interest, the number of modes retained in the transport method could be increased.

In addition to being accurate, the transport method is very fast in the Dozier-Tappert approximation. The PE results in Fig. 1 took on the order of a day of computer time to obtain, while the time required for the transport method was on the order of a minute. Therefore, the transport theory approach appears to be an attractive alternative to brute force Monte Carlo methods.

Figure 3 shows the comparison at short range between PE and transport theory for the total intensity. The high quality of the agreement in the water column is again evident, as are the differences in the sediment as discussed in reference to Fig. 2. Other approaches, such as ray tracing where the effect of boundary roughness is incorporated as a loss into a boundary reflection coefficient, may be capable of accurately modeling the coherent intensity [11]. However, we believe the ability to accurately and rapidly model the coherent plus incoherent intensity in the mid frequency region is new.

In addition to the average intensity, the fluctuations in the average intensity are of interest as the realization of surface roughness changes with time. One characterization of these fluctuations is given by the scintillation index, a fourth moment of the field. In principle, transport theory can be extended to obtain evolution equations for the fourth moments of the mode amplitudes. We have taken a simpler and more approximate approach. The total pressure is the sum of the coherent and incoherent pressure. If one assumes that the incoherent complex pressure obeys Gaussian statistics, it is straightforward to derive an expression for the scintillation index in terms of the coherent and total intensities. The result is

$$SI = \frac{< I >^2 - I_c^2}{< I >^2} = \frac{<| p |^2>^2 - |< p >|^4}{<| p |^2>^2}, \tag{3}$$

where $< p >$ and $<| p |^2>$ can be given by transport theory. Thus, we obtain a transport result for the scintillation index that can be compared with the PE result for the same quantity where no approximations (other than rough surface PE) are made. The comparison is shown in Fig. 4. To obtain good convergence to the fourth moments with PE simulations, the number of rough surface realizations was increased to 1,000 for this example. The agreement in the water column between transport theory and PE results for the scintillation index is very good, in spite of the approximations made.

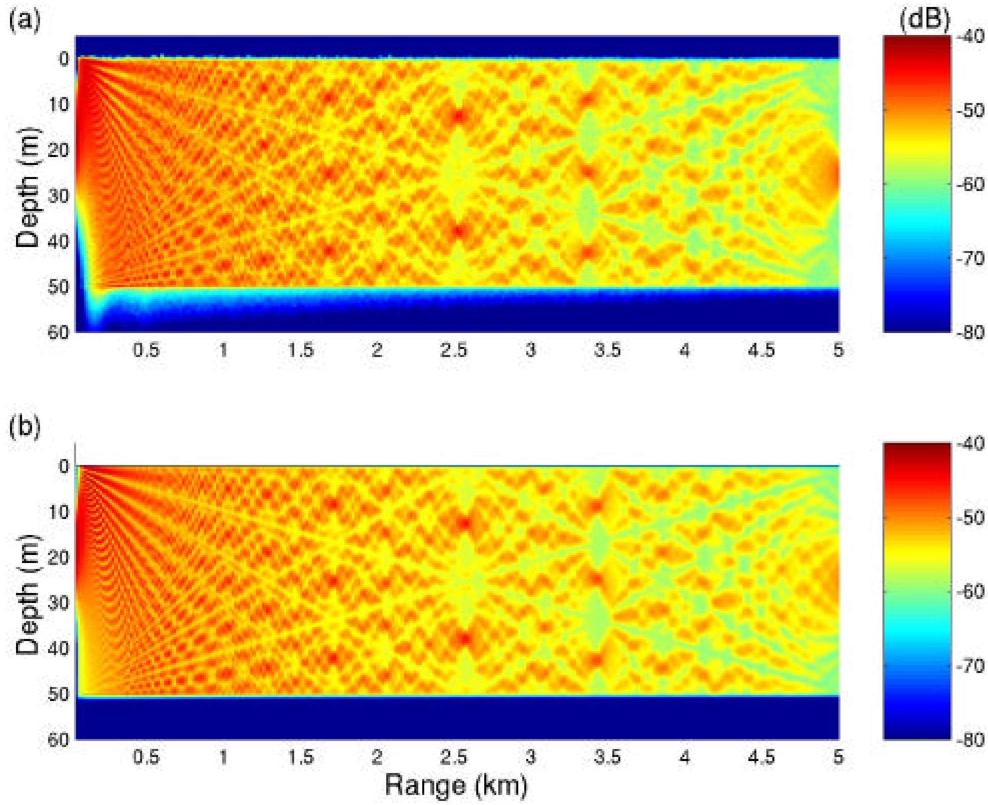

FIGURE 3. Comparison of (a) PE simulation for total intensity obtained by averaging over the results for 50 rough surface realizations with (b) transport theory result.

DISCUSSION

The results presented indicate that transport theory in the Dozier-Tappert approximation is a promising approach for modeling shallow water propagation at mid frequencies where boundary scattering can play an important role. The method as outlined is accurate and computationally fast for the problem studied. In addition to providing the coherent field and the total intensity, the scintillation index can be obtained with good accuracy at essentially no extra effort for the case treated.

The problem considered includes several significant simplifications. The geometry is two-dimensional, roughness is confined to the surface, sound speed variations (e.g., internal waves) and bathymetric variations are ignored. We believe that some, and perhaps all, of these restrictions can be relaxed in future work.

ACKNOWLEDGMENT

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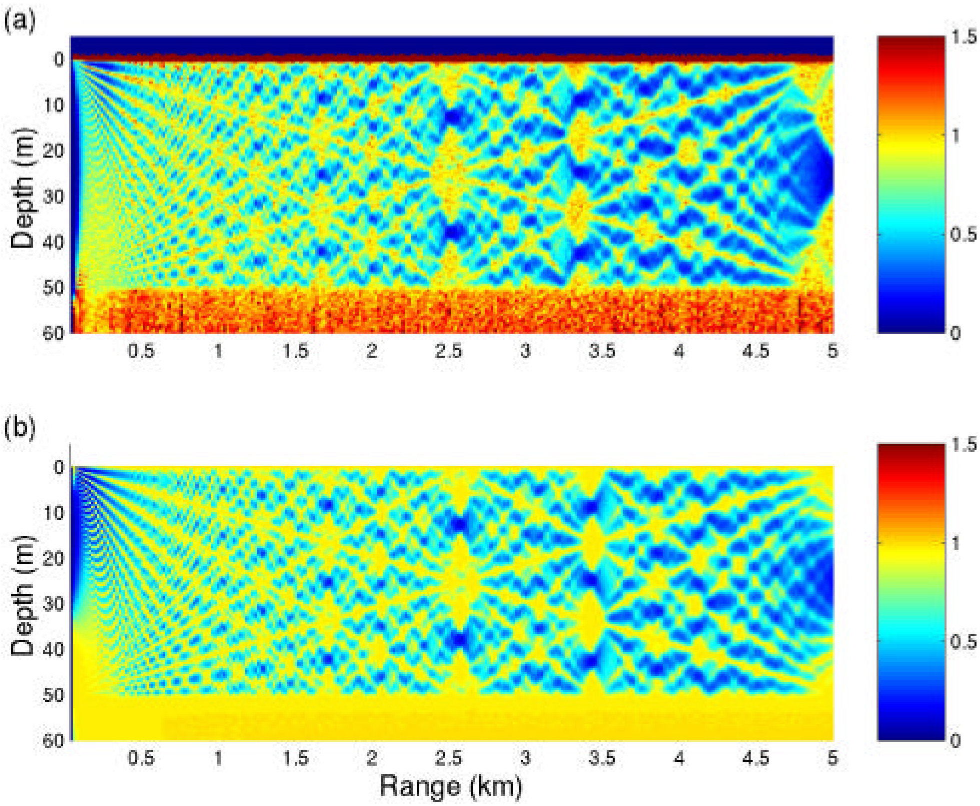

FIGURE 4. Comparison of (a) PE simulation for the scintillation index obtained by averaging over the results for 1000 rough surface realizations with (b) transport theory result.

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