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CHAOTIC EFFECTS IN MULTIPATH ENVIRONMENTS

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Models using the parabolic equation (PE) approximation are widely used for treating sound propagation in the ocean. Over the years extensive comparisons have been done and acoustic modelers have developed a sense of confidence about when such codes would be reliable. We were therefore startled to find grossly inaccurate PE results in a routine problem involving surface duct propagation. Curiously, certain older PE formulations were found to give accurate results. The physical and mathematical reasons for this are discussed.

Introduction

Surface ducts are a common feature in the world's oceans. They arise from wind-driven mixing and lead to a narrow waveguide within the broader waveguide formed by the ocean surface and bottom. The sound speed profile for our particular surface duct problem is shown in Fig. 1. The surface duct is seen in the upper 100 m where there is an upward refracting zone. Below that region we see a fairly typical deep water profile.

Such surface ducts can have important effects on acoustic propagation. Rays emanating from a source near the surface go through a cycle of refraction and reflection leading to a band of energy in the surface duct. Rays with steeper take-off angles escape the surface duct and refract up and down within the broader ocean waveguide. These features are illustrated in the ray trace in Fig. 1.

Parabolic Equation Results

The acoustic pressure is governed by the Helmholtz equation which in cylindrical coordinates reads:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} + k_0^2 n^2 p = 0. \tag{1}$$

Here $p(r,z)$ is the acoustic pressure, $k_0 = \omega/c_0$ is a reference wavenumber, and $n(r,z) = c_0/c(r,z)$ is the index of refraction. This elliptic equation can be converted to a variety of parabolic equations which are easier to solve numerically and whose solution provides an approximation to the original elliptic equation. The simplest such form is given by the standard parabolic equation (SPE):

$$\frac{\partial \psi}{\partial r} = \frac{ik_0}{2}\left(n^2 - 1 + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right)\psi. \tag{2}$$

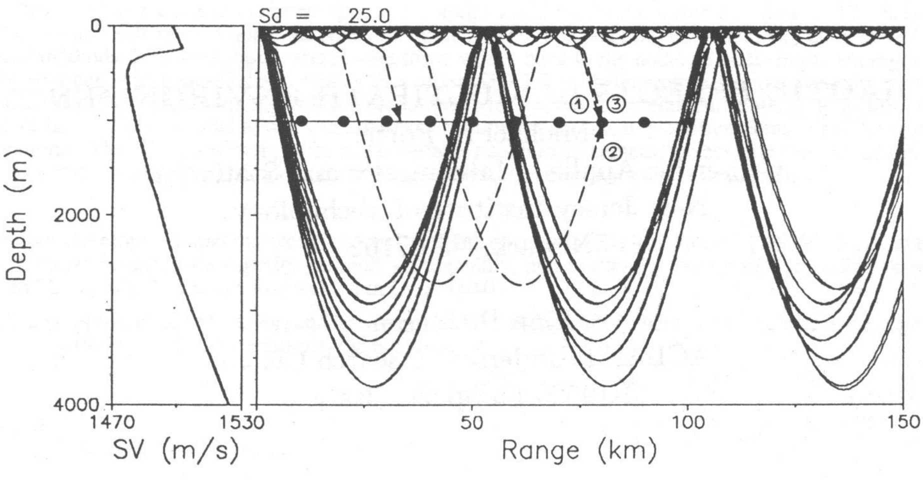

Figure 1: Sound speed profile and ray trace for the surface duct problem.

This equation is solved by standard numerical techniques and then the pressure is recovered via,

$$p(r,z) = A\psi(r,z)\frac{e^{ik_0 r}}{\sqrt{k_0 r}}, \tag{3}$$

where A is a constant.

The surface duct problem is range-independent and involves a fan of rays within ±10°. In short, it has the characteristics which would lead us to expect that a PE model would give accurate transmission loss predictions. For the PE calculation we selected a popular wide-angle formulation introduced to the ocean acoustics community by Thomson and Chapman[4]. The source was placed within the surface duct at a depth of 25 m and transmission loss was calculated for a receiver at a depth of 100 m. As seen in Fig. 2, the PE model (dashed line) showed enormous errors compared to a reference normal mode solution (solid line). The PE model correctly predicts the acoustic field out to the first convergence zone at 50 km. Beyond that range it gives the appearance of allowing the energy to leak out of the surface duct.

Curiously, the original and now seldom used formulation of the PE (the SPE[1] given in Eq. 2) yielded excellent results as seen in Fig. 3. Amongst the more modern formulations, the Claerbout PE used in IFD[5,6] also gave very accurate results, while the LOGPE[7] showed the anomalous drop-out beyond the first convergence zone.

In summary, this particular case, which seemed relatively benign at first, led to serious errors for some PE formulations while other PE's were unaffected.

Physical Interpretation

The source of the difficulties in modeling this problem becomes clear if we look at the arrival structure in detail. We replaced the single frequency source by a simple wavelet and synthesized the received field by running the normal mode model over a band of frequencies. By examining the received pulse at a number of different locations one obtains a fairly clear picture of the important paths. For example, consider a horizontal array of hydrophones located at a depth of 1000 m and extending from 10 to 100 km in range.

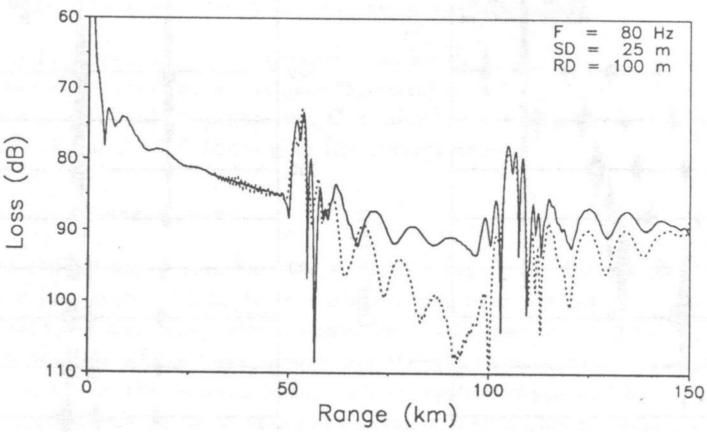

Figure 2: Transmission loss curve using the Thomson-Chapman PE.

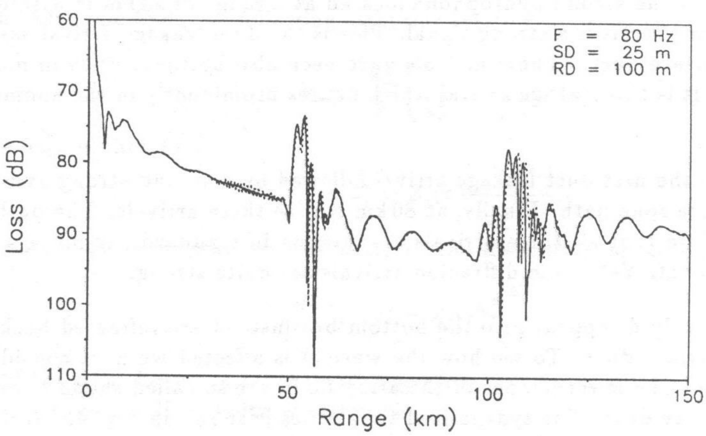

Figure 3: Transmission loss curve using the standard Tappert-Hardin PE.

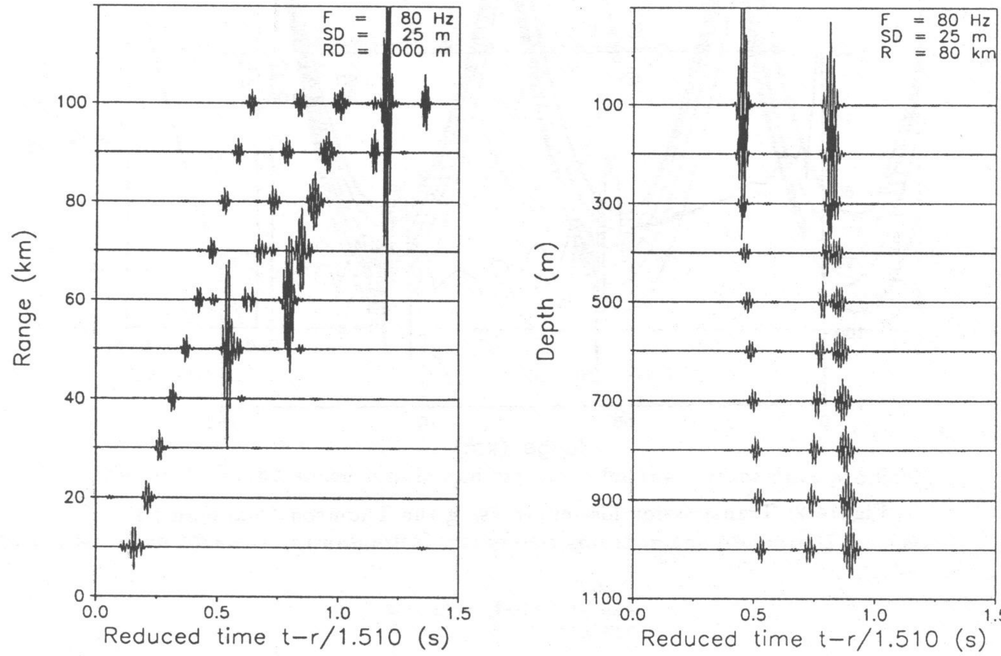

Figure 4: Arrival structure on an array a) horizontal, b) vertical.

The location of this horizontal array is indicated by the filled circles in Fig. 1. The received time series is plotted in Fig. 4a. Note that the second hydrophone located at a range of 20 km is within the shadow-zone. Nevertheless, the hydrophone receives a strong signal. This is the duct leakage arrival associated with a ray that diffracts out of the surface duct. (These arrivals have been also been observed in measurements at sea as discussed by Freese[9].) It is this leakage arrival which figures prominently in the anomalous transmission loss results.

At a range of 50 km we see the first duct leakage arrival followed by a second strong arrival associated with the deep cycling convergence zone path. Finally, at 80 km we see three arrivals. The paths of these arrivals are numbered in Fig. 1. Note that all three arrivals are missing in standard ray models since they involve leakage out of the surface duct. Yet, these diffracted arrivals are quite strong.

These leaky rays do not simply disappear into the bottom but instead are refracted back toward the ocean surface and re-enter the surface duct. To see how the wavelet is affected we next consider a *vertical* array located at a range of 80 km. The lower phones of the array lie in the so-called shadow zone while the upper phone is located in the surface duct. The synthesized time series is shown in Fig. 4b. Note that the deepest phone shows the same three arrivals we have already discussed. Indeed, this phone is located in the same position. As the receivers approach the surface duct, the leaky surface duct arrival gradually becomes a true ducted arrival and shows an increase in amplitude. The changes to the second and third arrivals are somewhat more interesting. The ray paths associated with these arrivals coalesce forming a 'leaky convergence zone' arrival with a higher amplitude than its components. Thus, at this range, a receiver in the surface duct sees not just the surface ducted arrival we referred to earlier, but a second leaky CZ arrival which is comparable in strength to the ducted arrival. This second arrival can only occur beyond the first convergence zone.

In summary, the range from about 50 to 100 km, the field in the surface duct is derived from the constructive or destructive interference of two dominant arrivals. Their relative phase is critical in determining the transmission loss. Indeed, we find that by simply changing the mean sound speed in the surface duct by 0.5 m/s we can change the transmission loss by about 10 dB. The problem itself is therefore intrinsically sensitive without regard to any errors in the numerical calculations.

It is also clear that for reasonable transmission loss results, any numerical code must be able to propagate energy along the two dominant ray paths over the roughly 1000 wavelengths in range while preserving their relative phase. This is a difficult requirement. Considering that phase errors are a key problem in PE models it is surprising that any of the PE codes give the correct answer.

Parabolic Equation Phase Errors

A central question still remains unanswered: why do some of the PE models work and others fail on this problem? We have just seen that there is a difficult requirement for phase accuracy. The answer lies in the fact that the trapped and leaky arrivals involve nearly identical ray take-off angles but quite different trajectories. Those models whose phase errors are strictly dependent on take-off angle will therefore make very similar phase errors in the two ray paths. Their *relative* phase will be preserved. Those models whose phase error has a dependence on the actual path taken will scramble the relative phase of the two ray paths.

We shall briefly demonstrate that the standard PE given in Eq. (2) has the desired property that the errors can be understood in terms of the take-off angle alone. First we review the result of McDaniel[8] which relates phase errors in modes of the standard PE (SPE) to those of the original Helmholtz equation. We seek a solution of the SPE in the form:

$$\psi(r,z) = Z_j(z)e^{i\kappa r}, \tag{4}$$

where Z_j is an eigenfunction of the depth-separated equation

$$\frac{d^2}{dz^2}Z_m(z) + k_0^2 n^2 = k_m^2 Z_m(z). \tag{5}$$

Substituting in the SPE one finds the following algebraic relation between the horizontal wavenumber κ and the wavenumber k:

$$\kappa_m = \frac{k_0}{2}\left(\frac{k_m^2}{k_0^2} - 1\right). \tag{6}$$

The SPE pressure field is therefore

$$p_j^{SPE}(r,z) = Z_j(z)\frac{e^{i\kappa r}e^{ik_0 r}}{\sqrt{r}}. \tag{7}$$

This may be compared to the normal mode solution to the Helmholtz equation:

$$p_j^{Helm}(r,z) = Z_j(z)\frac{e^{ik_j r}}{\sqrt{r}}. \tag{8}$$

Thus the modes of the SPE have the same depth eigenfunction but a different phase dependence in range compared to the modes of the Helmholtz equation[8].

Now we can explain why these phase errors do not seriously degrade the surface duct problem. Note that the two dominant paths in the surface duct involve ray take-off angles that are nearly identical: the true ducted arrival has a very low angle so as to remain in the duct while the leaky arrival has a take-off angle that is just large enough to allow the ray to graze to bottom of the duct. We have just seen that a single spectral component of the PE may have serious phase errors however both these arrivals are associated with the same spectral component and therefore have the same phase error.

The PE models that fail do not have this property (that the eigenfunctions of the elliptic problem are also eigenfunctions of the Helmholtz problem). Instead, their phase errors depend not just on the take-off angle of a ray but on the particular path of the ray. Thus the leaky ray which refracts through the lower part of the water column inherits different phase errors from the ray with a nearly identical take-off angle that spends its time in the surface duct.

Conclusion

This simple surface duct problem manifests a number of interesting features. Duct leakage arrivals which are often overlooked turn out to be quite strong in amplitude. Furthermore, these arrivals are refracted through the water column and refocus in the surface duct. The erratic behavior of certain PE models is seen to result from errors in the relative phase of these arrivals in the surface duct.

One could simply stop using the models that are sensitive to this problem; however, some— such as the the Thomson-Chapman formulation —have important advantages for many applications. There is therefore a need for an algorithmic modification to correct this problem.

While this case raises some interesting numerical issues there are practical questions which may be of greater importance— given that the problem itself shows extreme sensitivity to the environmental data and given that the environment is usually not known very precisely, are the results of any model physically meaningful? The answer to this question depends on how the transmission loss data is being used. Often a 'typical' result is desired. For instance, we may select a frequency in the band of interest or a single sound speed profile in the time varying ensemble. This surface duct case is an example where a single realization is not representative of the mean.

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