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Coupled Modes for Rapid Range-Dependent Modeling

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Abstract: Normal mode models are widely used for solving range-independent ocean acoustic problems. The approach generalizes to range-dependent problems by dividing the problem into a sequence of range-independent segments and using normal modes to represent the solution in each segment. While this coupled-mode approach has proven extremely useful for checking other models, it has generally been considered uncompetitive with parabolic equation (PE) algorithms in terms of run time.

We show that an optimized coupled-mode algorithm is practical and in fact competitive with the PE. To develop an efficient algorithm, we take advantage of a widely used finite-difference algorithm for solving the range-independent normal mode problem. As in PE models, we make the *a priori* assumption that the field is dominated by the outgoing component. We also bypass the calculation of mode coupling matrices, and compute the mode amplitudes in a new segment directly by projecting the pressure field onto the new mode set. This allows the solution to be constructed by a simple marching. We illustrate the algorithm using several oceanic scenarios involving range-dependent oceanographic and bathymetric features.

1. INTRODUCTION

A common starting point for ocean acoustic problems is the Helmholtz equation in two-dimensions:

$$\frac{1}{r}\frac{\partial p}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \rho(z)\frac{\partial}{\partial z}\left(\frac{1}{\rho(z)}\frac{\partial p}{\partial z}\right) + \frac{\omega^2}{c^2(r,z)}p = \frac{-\delta(z-z_s)\delta(r)}{2\pi r}. \tag{1.1}$$

where ρ is the density, $c(r,z)$ the sound speed, and ω is the circular frequency of the source. This equation must also be augmented with appropriate boundary conditions.

One way of solving the Helmholtz equation is to divide the problem into a sequence of N range-independent segments in range [1] as illustrated in Figure 1. Then, within each range-segment the exact solution can be constructed using normal modes as a sum of right- and left-traveling waves. Neglecting contributions form higher-order modes or from the continuous spectrum, the general solution in the jth segment can be written as follows:

$$p^j(r,z) = \sum_{m=1}^{M}\left[a_m^j H1_m^j(r) + b_m^j H2_m^j(r)\right]Z_m^j(z), \tag{1.2}$$

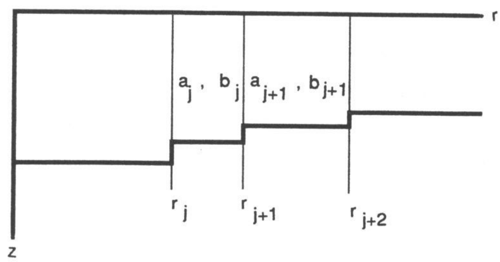

Fig. 1. Segmentation for the coupled mode formulation.

where $H1,2$ are the following ratios of Hankel functions,

$$H1^j_m(r) = \frac{H_0^{(1)}(k^j_m r)}{H_0^{(1)}(k^j_m r_{j-1})}, \tag{1.3}$$

$$H2^j_m(r) = \frac{H_0^{(2)}(k^j_m r)}{H_0^{(2)}(k^j_m r_{j-1})}, \tag{1.4}$$

and k^j_m *and* $Z^j_m(z)$ are solutions of the depth-separated equation:

$$\rho(z)\frac{d}{dz}\left(\frac{1}{\rho(z)}\frac{dZ(z)}{dz}\right) + \left(\frac{\omega^2}{c^2(z)} - k^2\right)Z(z) = 0,$$

$$Z(0) = 0,$$

$$\frac{dZ}{dz}(D) = 0) \tag{1.5}$$

The boundary conditions imposed imply a pressure release surface located at $z = 0$ and a perfectly rigid bottom located at $z = D$. Then, imposing continuity of pressure and particle velocity along each of the vertical interfaces leads to a large block-banded matrix for the mode coefficients a^j_m and b^j_m in each segment. The pressure field is then computed by summing up the modes in each segment as given in Eq. (1.2).

This type of approach has been used in the COUPLE model [1] and successfully applied to a number of benchmark problems [2,3]. While extremely useful for providing benchmark solutions, the direct solution of the Helmholtz equation in this manner is usually not practical for ocean acoustic problems because of the execution time.

Instead, for range-dependent problems the method of choice is often a PE type solution. This approach takes advantage of the fact that ocean acoustics problems are often dominated by just the right-traveling component of the solution, leading to equations which can be rapidly solved by marching forward in range.

Our objective in this paper is to examine the alternative of using coupled-modes in a similar one-way fashion. The key question is whether or not an optimized marching solution based on normal modes is competitive in terms of run-time with existing PE models.

2. ONE-WAY COUPLED NORMAL MODES

To obtain the one-way formulation in each segment we seek a solution in the form of just a right-traveling wave:

$$p^j(r,z) = \sum_{m=1}^{M} a^j_m Z^j_m(z) H1^j_m(r). \tag{2.1}$$

With the range of possible solutions restricted to just the right-traveling component, we must relax the continuity conditions on vertical interfaces. For the sake of comparison with existing models we have implemented the one-way coupled mode solution using pressure-matching at interfaces discarding the condition of continuity of particle velocity. However, as discussed in Ref. [3] this informal step can have important implications for solution accuracy. A single-scatter type condition is just as easy to implement and yields much improved results. Therefore, it is normally our method of choice.

The condition of continuity of pressure at each interface can be written:

$$\sum_{m=1}^{M} a^{j+1}_m Z^{j+1}_m(z) = \sum_{m=1}^{M} a^j_m H1^j_m Z^j_m(z). \tag{2.2}$$

where we have used the fact that $H1^{j+1}_m(r_j) = 1$. Taking advantage of the mode orthogonality, we apply the operator:

$$\int (\cdot) \frac{Z_l^{j+1}(z)}{\rho_{j+1}(z)} dz, \tag{2.3}$$

yielding:

$$a^{j+1}_{\ell} = \sum_{m=1}^{M} a^j_m H1^j_m(r_j) c^j_{lm}, \quad l = 1, \ldots, M, \tag{2.4}$$

where,

$$c^j_{lm} = \int \frac{Z_l^{\,j+1}(z) Z_m^{\,j}(z)}{\rho_{j+1}(z)} dz, \tag{2.5}$$

In matrix form, Eq. (2.4) can be written:

$$\mathbf{a}^{j+1} = \mathbf{C}^{j}\mathbf{H}_1^{j}\mathbf{a}^{j}. \tag{2.6}$$

Well-polished codes exist for solving for the normal modes in quite complicated multilayered environments. A popular technique uses a standard centered finite-difference approximation combined with Richardson extrapolation [4]. The resulting tridiagonal algebraic eigenvalue problem is solved using roughly $30MN_z$ floating-point operations where N_z is the number of grid points in depth and M is the number of modes calculated. Typically, $M \approx N_z/10$ so that we obtain an operation count of $3N_z^2$. An algorithm for doing the one-way mode coupling has been incorporated in two popular implementations of the modal algorithm (KRAKEN [5] and SNAP [6]). We refer to the one-way coupled-mode version of SNAP as C-SNAP and will be showing results obtained with that model. Timings and results are similar for the coupled mode version of KRAKEN.

The modes calculated by these models are provided on a finely tabulated grid of depth points. The mth vector is then used to define the mth column of a matrix $\mathbf{U}^{j}$. For an isodensity problem, we can then approximate the coupling matrix by the discrete form

$$\mathbf{C}^{j} \approx (\mathbf{U}^{j+1})^{t}\, \mathbf{U}^{j}, \tag{2.7}$$

which is equivalent to evaluating the coupling integral by the trapezoidal rule. (For a variable density problem this equation is slightly modified.) Substituting in Eq. (2.6) we obtain:

$$\mathbf{a}^{j+1} \approx \left\{(\mathbf{U}^{j+1})^{t} \left[\mathbf{U}^{j}(\mathbf{H}_1^{j}\mathbf{a}^{j})\right]\right\} \tag{2.8}$$

We can describe the steps in this equation as follows: one advances the phase of the coefficients to the next segment, then one sums up the modes to compute the field just to the left of the interface and, finally, one projects the pressure field onto the mode set in the next segment. Computing the coupling matrix would involve the calculation of the matrix-matrix product $(\mathbf{U}^{j+1})^{t}\mathbf{U}^{j}$, but when the operations are done in the order indicated by Eq. (2.8), one performs only the operation of a matrix times a vector and, therefore, obtains a significant savings in execution time.

Let us consider the alternative PE approach. The pressure field is represented in terms of an envelop function as

$$p(x,z) = \psi(x,z)e^{ik_0 x} \tag{2.9}$$

where the envelope function then satisfies

$$\frac{\partial \psi}{\partial r} = \frac{ik_0}{2}\left(n^2 - 1 + \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right)\psi. \tag{2.10}$$

This is the standard PE originally considered by Tappert [7]. In fact, we shall be presenting comparisons with a popular implicit finite-difference PE (IFDPE) [8] which used a higher-order approximation to the square root operator. The PE equation is then discretized using a simple centered-finite difference operator. A single range-step then requires solving a linear system and doing a matrix-vector multiply. The matrices involved are all tridiagonal so that about $10N_z$ floating-point operators per range step are required, where N_z is the number of points in the depth grid. The complete solution is, thus, calculated in $10N_zN_r$ operators where N_r is the number of range steps.

When the normal mode problem is solved using the same centered finite-difference approximation, the cost of computing the modes at a single range is roughly $3N_z^2$ operations. Thus, if N_{prof} stairsteps are required to define the environment, the total cost is $3N_z^2N_{prof}$ operations. (The final mode synthesis typically uses a small percentage of additional time, although in cases where many source/receiver combinations are involved, it can dominate.)

Thus, we have

$$10N_zN_r \qquad \text{IFDPE} \tag{2.11}$$

$$3N_z^2N_{prof} \qquad \text{Normal Mode.} \tag{2.12}$$

For range-independent environments ($N_{prof} = 1$) where the field is desired beyond a few water depth $N_r > N_z$ the normal mode solution is faster. For this reason, normal mode solutions have generally been favored for range-independent environments.

However, for range-dependent environments the normal mode calculation must be done in each segment where the environment is updated. For gradually varying environments, a few updates suffice and the normal mode approach is significantly faster. For strongly varying environments, it may be necessary to use a new profile every wavelength in range. In this case, the standard PE approach is significantly faster.

The question then is: Do typical ocean environments vary enough to favor PE solutions or coupled normal mode solutions? We can give a partial answer by considering some example problems.

3. PROPAGATION OVER A SEAMOUNT

The environment is illustrated schematically in Figure 2. The sound speed profile is a canonical deep water profile. Range-dependence in the problem comes from an idealized seamount that is symmetric and extends from 80 km to 120 km and is 1000 m high. This feature was modeled using C-SNAP with approximately 100 range-independent segments. The resulting transmission loss is shown in Figure 3 for a source depth of 100 m and a source frequency of 50 Hz. In order to obtain a solution which is sufficiently narrow-angled so that the PE solution is valid, we have used a modal starting field retaining only those modes which are waterborne; that is, turned before hitting the bottom. The field shows a convergence zone type pattern involving a beam of energy cycling up and down the water column. At a range of about 90 km the beam hits the seamount and reflects at steeper angles.

We have also solved this problem using the IFDPE model and obtained a plot that is visually indistinguishable from the coupled-mode result in Figure 3. A more quantitative sense of the error is seen by comparing a slice form the transmission loss plot taken at a receiver depth of 300 m as shown in Figure 4. We observe that there is excellent agreement between the IFDPE and one-way coupled-mode solutions. The execution time for both models is approximately half an hour on a roughly 1 megaflop machine (VAX 8600).

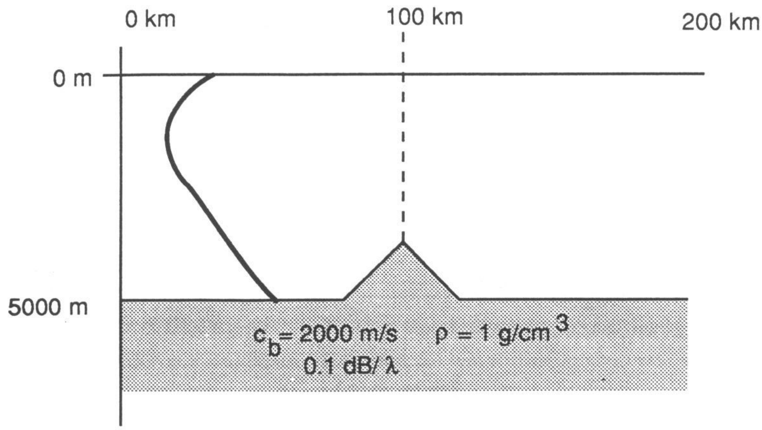

Fig. 2. Schematic of the seamount problem.

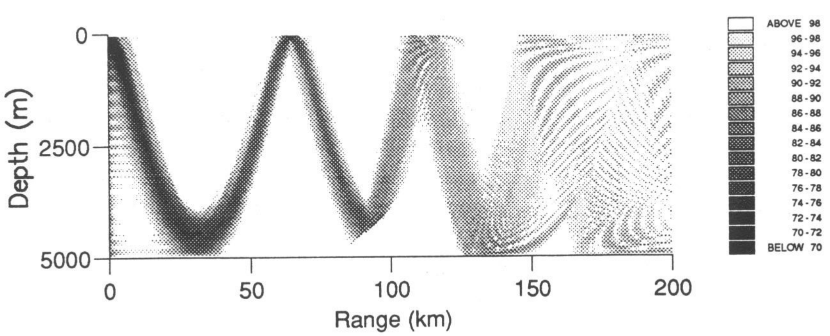

Fig. 3. Coupled mode transmission loss for the seamount problem.

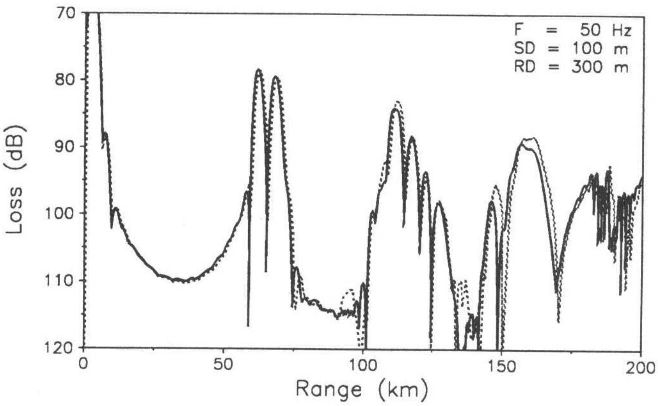

Fig. 4. Transmission loss for the seamount problem at a receiver depth of 300 m (C-SNAP (—), IFDPE (- - -)).

4. PROPAGATION OVER A CONTINENTAL SLOPE

A schematic of the environment is shown in Figure 5. This type of environment is a prototype of continental slope propagation where the initial 500 m in range represents a continental shelf. (This problem is a modified version of bench mark problem 5 from the PE-II workshop [9] with the sediment layer removed to obtain a problem solvable by the COUPLE program.)

Taking a source depth of 100 m and the source frequency of 25 Hz we obtain with C-SNAP the transmission loss shown in Figure 6. (Approximately 30 range-independent segments were used in this calculation.) Again, in order to accommodate the angle limitations of the PE, we have chosen to use a narrow-angle source generated using just the discrete modes at the origin. The field shows a somewhat complicated 4-8 mode interference interference pattern. Once again, the IFDPE results (not shown) were indistinguishable in terms of the grey shade plot.

Again to be precise about the level of agreement between the models, we turn to a line plot taken at a fixed receiver depth of 150 m. This problem involves few modes so that we can also provide an independent check using the full two-way coupled-mode solution (COUPLE). The comparison of C-SNAP, IFDPE, and COUPLE is shown in Figure 7 showing excellent agreement between all three models. As in the previous test problem, the one-way coupled-mode (C-SNAP) and IFDPE solutions required comparable times (approximately 1 minute).

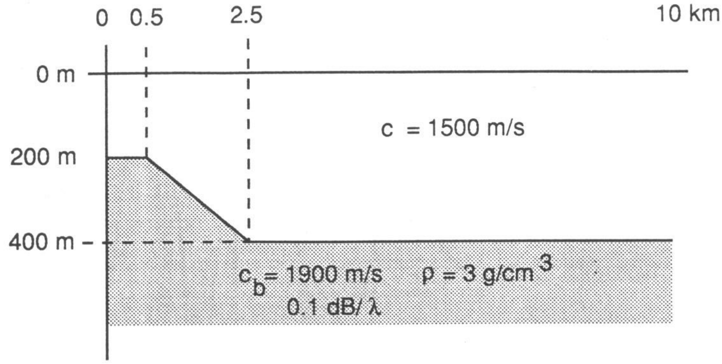

Fig. 5. Schematic of the continental slope problem.

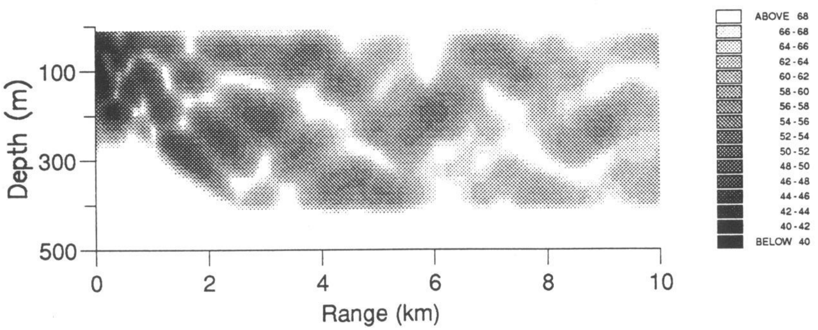

Fig. 6. Transmission loss for the continental slope problem.

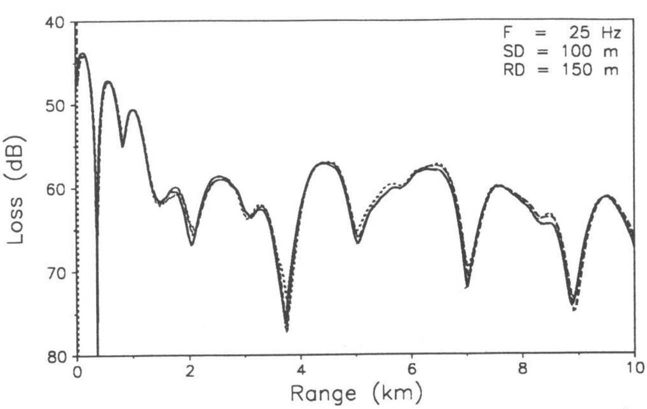

Fig. 7. Transmission loss for the continental slope problem at a receiver depth of 150 m [C-SNAP (—), IFDPE(- - -), COUPLE (– · – ·)].

5. SUMMARY AND CONCLUSIONS

We have shown that the normal mode approach offers a viable alternative to PE modeling for range-dependent environments with run-times which, in our test problems, are roughly comparable. A precise comparison of execution time is complicated by the fact that it is difficult to define an error criterion which all would agree is meaningful to the user. Which is more important, accuracy in the convergence zone position or in its level? Furthermore, there are a number of parameters in each model which can be tuned to optimize execution time (e.g., source spectrum, range and depth grids, angular width of the PE or spectrum).

In favor of the coupled normal mode approach, we note that while work has been done to improve PE's, little work has been done on optimizing coupled normal modes and, indeed, there seem to be numerous possibilities for further improvement. For instance, generalized "wedge-modes" [10] may allow for much larger range-steps.

Furthermore, the normal mode approach allows multiple source depths to be handled with negligible additional effort since execution time is dominated by the time required to compute the modes, a calculation which does not need to be repeated for additional depths. This benefit is particularly important when matched-field processing is used to localize sources by scanning over source position (see Ref. [11] and references therein).

It seems probable that there will always be a place for PE models. Circumstances favoring their use include 1) problems with range-dependence among the entire track and 2) problems where the field is desired on a fine range-depth grid. However, we feel that one-way coupled mode algorithms offer many possibilities, and suitably optimized may well prove a more desirable alternative for many ocean acoustic problems.

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