# MODEL VALIDATION FOR DIRECT AND INVERSE PROBLEMS

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ABSTRACT Sophisticated acoustic models form the inner loop in most inversion schemes. Trust in these models has come from comparing their results to scale-model and at-sea experiments. The modeling is challenging, partly because ocean acoustic problems span a wide range of frequencies. At the high end we have sources squealing at megahertz frequencies, propagating sound just a few millimeters. This is the scenario for sing-around velocimeters used to measure the speed of sound in the ocean. At the low end sources rumble like the deepest organ notes at tens of Hertz making sound heard around the world. Such problems emerge for a new type of sing-around velocimeter proposed for monitoring global warming. Important decisions are made based on the models. Can they be trusted? We briefly review some of the efforts that have been made to answer this question.

## 1. Introduction

As research in ocean acoustics has changed, the demands on the models have changed accordingly. The models are applied to new areas and must be validated anew. The earliest benchmarking efforts related to the once dominant ray models. Later, parabolic equation (PE) models emerged and were compared in the first PE workshop[1]. Initially the focus was on range-independent applications. Later the much more challenging range-dependent problems were studied in depth[2, 3, 4]. Careful tests have also been done to compare blind acoustic-model predictions to at-sea experiments[5].

The problem of knowing the environment is often cited as the biggest problem for acoustic modeling. Indeed it is often argued that the models are now so accurate that further improvements are not worthwhile. However, as the papers in this volume show there has been much progress on the important problem of using sound to sense the environment. Furthermore, many inversion schemes work by repeatedly solving the forward problem as the environment is brought into focus: If the acoustic models are inaccurate then they are again the limiting factor.

There have been few efforts to do careful performance comparisons of different algorithms for inverse problems. However, in a way the process is easier. In benchmarking solutions to forward problems the researcher often has the opportunity to do his own intermodel comparisons to find the 'exact' solution. He may then adjust 'knobs' in a program to get the best fit. This has been an issue in both early PE studies (where the reference sound speed was adjusted) and in Gaussian beam models (where the initial beam width and curvature could be tuned).

However, for inversion schemes one may give the data required for an inversion without the solution. The data may also be corrupted by noise to provide a test of the robustness of the inversion scheme. This approach was used very successfully in a recent matched-field

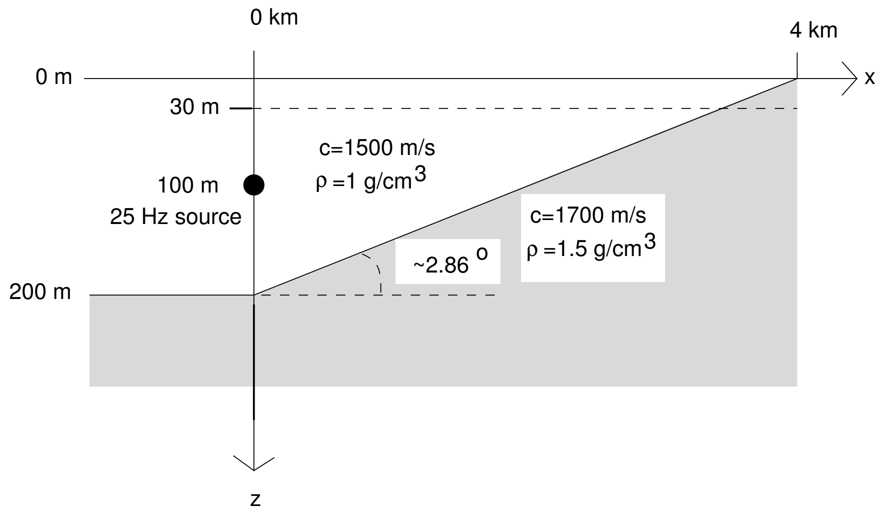


FIGURE 1. Schematic of the wedge problem.

processing workshop[6] to compare different approaches to the inverse problem of finding the source position.

In the following sections we shall review a couple of interesting benchmarking problems. Typically these sorts of reviews focus on the successes. Here instead, we shall examine the notable failures and the lessons learned from them. For details on the models approaches we refer the reader to Ref. [7].

## 2. The Wedge

The problem of sound propagating in a wedge is one of a few canonical problems in underwater acoustics which have been studied in depth. When the boundaries of the wedge are perfectly reflecting the problem may be solved analytically. In 1980 Jensen and Kuperman[8] used the PE to study propagation in a wedge-shaped ocean with a *penetrable* bottom. This problem shows some very interesting physical phenomena including the cutoff of a mode propagating upslope and the resulting injection of a beam into the ocean bottom.

A version of the penetrable wedge problem was chosen for the ASA benchmark tests in 1990. The particular scenario is shown in Fig. 1 Some very curious results emerged from those comparisons as shown in Figure 2. If we look first at the comparison between one-way coupled-mode and PE results we see excellent agreement— the lines are almost indistinguishable on the plot. One tends to conclude from this that both models are correct. However, the opposite is true! The correct answer is obtained by a full two-way coupled mode calculation, that is, one that allows for back-scatter. This is shown as the solid line in Fig. 2.

Initially this discrepancy was attributed to back-scatter. However, a ray model[9] gave the correct result without backscatter. Furthermore some PE's gave very excellent results.

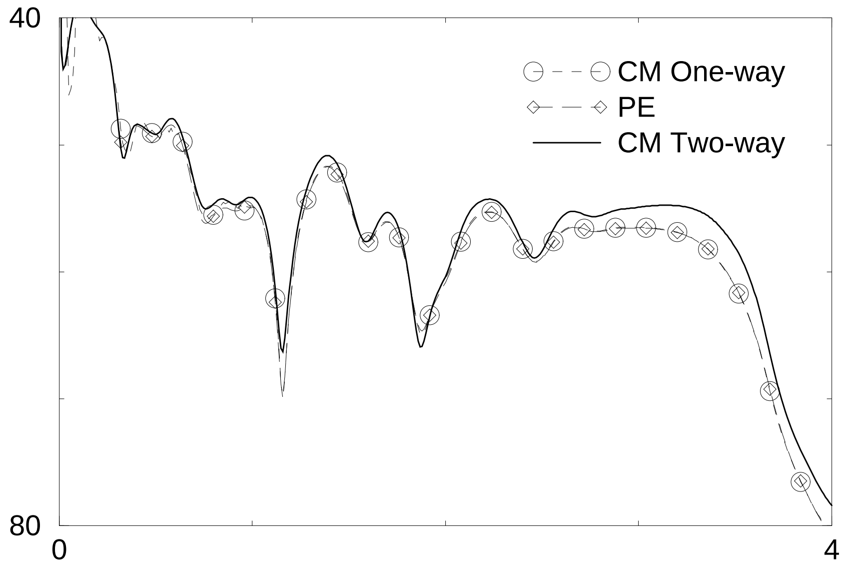


FIGURE 2. Coupled-mode and PE results for the wedge.

Lastly, it was found that when the back-scattered component was extracted from the full two-way coupled-mode solution, it was negligible.

Thus, it seemed it should be possible to construct a one-way model that gave the exact outgoing solution. On the other hand, none of the mode or PE models were doing this well. The models that did poorly all used a stairstep approximation to the bottom and were losing a small amount of energy at each step. Over range these errors accumulated and became significant. Fortunately, a simple correction was all that was required to resolve these problems in both PE and coupled mode codes[10, 11].

## 3. The Leaky Surface Duct

Another unexpected failure occurred in the leaky duct problem[12] shown in Fig. 3. This became Test Case 7 in the second PE workshop. In Fig. 4 we plot the TL source at 25 m and a receiver at 100 m depth (both in the surface duct). The source frequency is 80 Hz. The exact solution is given by a normal mode code yielding the solid line. Considering the narrow angle of propagation and the gradual variation in transmission loss, one expects that a PE should give a nearly perfect answer. To be still more certain, we initially used a modern wide-angle PE due to Thomson and Chapman.

Curiously, the Thomson-Chapman PE did extremely well out to the first convergence zone (CZ) at 50 km. Beyond that it looks as if perhaps something is triggered in the PE causing it to dump energy out of the surface duct. Then, after the second CZ the levels are close to the true values again. When the problem was rerun using the original PE developed by Tappert in 1973 (and now seldom used) excellent results were obtained, adding further to the confusion.

Through a careful broadband study the cause of this problem emerged. We tend to think

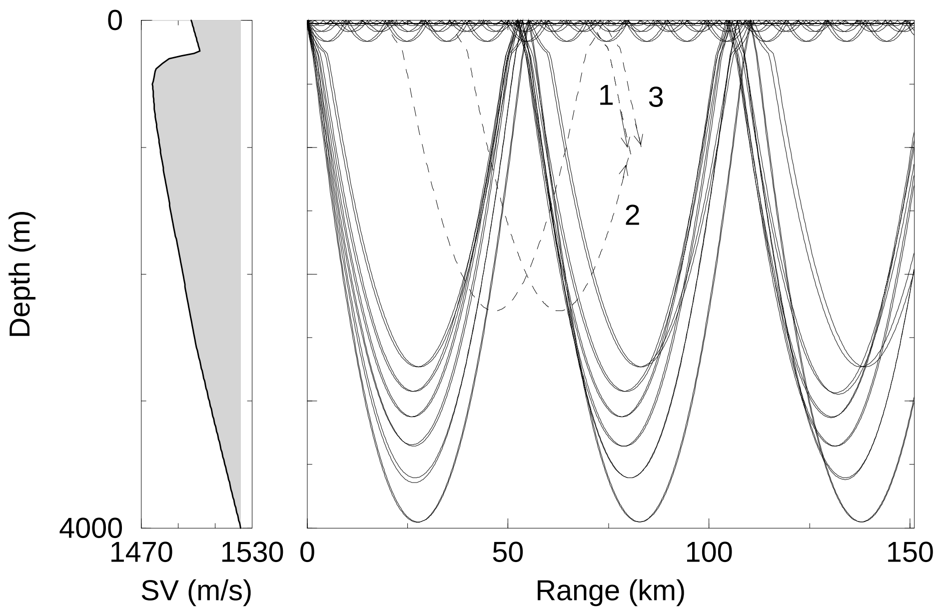


FIGURE 3. Sound speed profile and corresponding ray trace for the surface-duct problem.

of the field in the surface duct as dominated by ray paths that are trapped in the surface duct. Here, it turned out that ray paths associated with leakage out of the duct were strong. (These paths are shown by the dashed lines in Fig. 3.) Beyond the first CZ these leakage paths refocus in the surface duct and interfere constructively or destructively with the ducted paths. An error in the predicted path-length of just half a wavelength is sufficient to cause destructive interference and thus the dropout between the CZ's. All the PE's make such small errors in the phase of the various paths; however, some PE's made consistent errors for the interfering ray paths and therefore preserved the *relative* phase. This explains the fact that some PE's worked well and others failed.

The Thomson-Chapman PE used here has many useful properties. As such, work is ongoing to produce a modified version that is immune to this problem. However, there is a vastly more important aspect to this test problem: it is intrinsically ill-posed in the sense that the TL is extremely sensitive to small changes in the environment. A change of just 0.1 ms/s in the mean sound speed in the duct is sufficient to cause a similar dropout. We may think of this environment as just one snapshot in a time-evolving ocean. Then we must conclude that a TL calculation from a single realization is not meaningful.

## 4. Conclusions

In the last few years the applications have changed significantly. For instance, interest in acoustic transients has led to an increased emphasis on broadband modeling[13]. Similarly, a renewed interest in mine countermeasures has prompted work in the very-high frequency

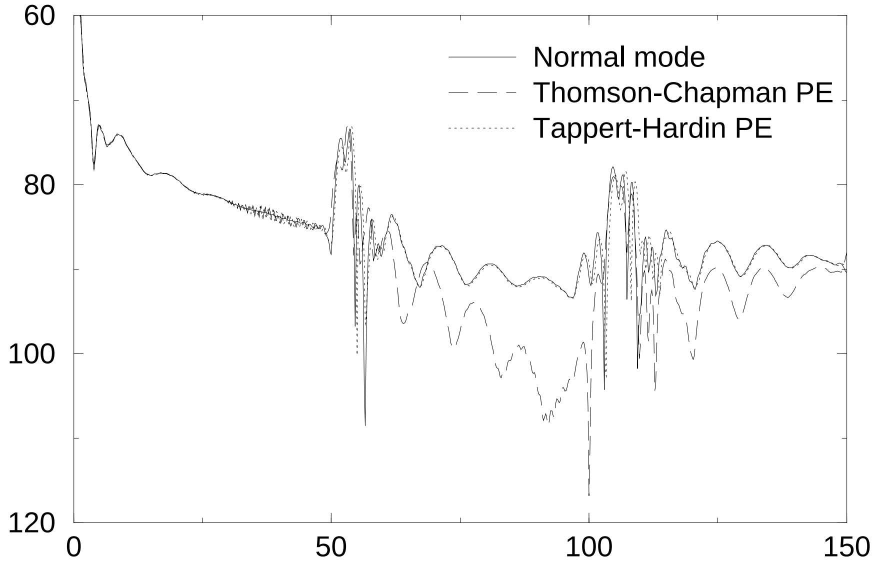


FIGURE 4. Comparison of mode and PE solutions for the surface-duct problem.

regime.

For years, reverberation modeling was dominated by heuristic generalizations of ray-based models. However, lately more formal full-wave approaches have become practical, thus motivating another recent workshop[14]. Lastly, sound is now being contemplated as a tool to probe the ocean on a transoceanic scale to monitor global warming[15, 16]. To date there have been few systematic intermodel comparisons over such distances[17].

In response to these challenges, acoustic models have continued to improve over time. Of course, the models have also benefited greatly from the faster heartbeat of modern computers. Not surprisingly, demands are being placed on them to match their increased capability.

It may be argued that the models are now mature: for fleet applications it is true that details of the environment often affect the results much more than numerical errors in the models. On the other hand, the models themselves will be used to probe the environment, for instance in acoustic tomography. In such applications one envisions acoustic models run across a sweep of frequencies for 3D environments. Furthermore, the models will be run repeatedly as the environmental is brought into focus. These sorts of problems are the ‘grand challenges’ for acoustic modelers in the next decade.

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